Mechanical model for the plasma maser effect

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(Received 21 December 2000; published 23 May 2001)

A Fermi-like pinball model is proposed for the nonlinear plasma maser effect, or turbulent bremsstrahlung, in the nonlinear interaction of plasma particles and waves. The model consists of a system of many noninteracting particles bouncing elastically between two oscillating walls. The walls act as energy and momentum sources and sinks for the particles, analogous to the wave fields in a weakly turbulent plasma. The oscillation amplitudes and frequencies of the walls determine the dynamics and distribution of the particles. The resulting asymptotic velocity distributions agree qualitatively with existing weak turbulence theories. It is also found that the second wall, which simulates the effect of the nonresonant wave-particle interaction, can destroy correlations in the particle dynamics and lead to the formation of a high-energy tail in the velocity distribution.

DOI: 10.1103/PhysRevE.63.067401 PACS number(s): 52.35.Ra, 52.25.Gj, 52.35.Mw

I. INTRODUCTION

Fermi [1] proposed that the motion of a particle bouncing between a fixed and an oscillating wall could be a model for the acceleration of cosmic rays to very high energies. However, it was later found that indefinite acceleration, or heating, of the particle does not occur in general and there exists an adiabatic limit. In fact, the particle motion is chaotic at low energies. At higher energies (velocities) its phase space exhibits a fractal structure, exhibiting features such as stochastic islands and invariant curves with narrow isolated stochastic layers [2]. Thus the original expectation that the particle could be stochastically accelerated to very high energies by this mechanism cannot be realized except for certain special cases.

Applications of Fermi-like mechanisms to collisionless, anomalous, or stochastic electron heating in laboratory and industrial plasmas have also been proposed [3]. In plasmas, electrons can be collisionlessly heated by repeated interactions with localized electric or magnetic field oscillations associated with collective plasma modes [4–6]. The latter are randomly distributed in space if the plasma is homogeneous and weakly turbulent. Such collisionless heating has been found in rf capacitive discharges, microwave electron cyclotron resonance discharges, as well as rf inductive discharges. It is also closely associated with the anomalous high-frequency skin resistance in metals at low temperatures [7]. When using the Fermi mechanism to model such processes, the prescribed oscillations of the wall simulate the fields of the normal (self-consistent) modes in the plasma. Consistent with the quasilinear approach, the self-consistent response of the waves to the redistribution of the momentum and energy of the particles is neglected. Thus the model may not be applicable if the wave-particle interaction is highly nonlinear.

In this paper we propose that a suitably generalized Fermi mechanism can be a simple mechanical model not only for quasilinear particle energy redistribution but also for the plasma-maser, or turbulent bremsstrahlung, effect [8–10] in a weakly turbulent plasma. The origin of the latter effect is the interaction of the nonresonant particles with the resonant and nonresonant waves in a weakly turbulent wave-plasma system [8,9]. The contribution of the nonresonant particles, usually ignored in the standard quasilinear treatment, turns out to be crucial for global energy and momentum conservation, and can affect the overall evolution of the system [5,6,8,9]. Here, resonant waves are waves satisfying the Cherenkov wave-particle resonance condition $\omega-\mathbf{k}\cdot\mathbf{v}=0$, where ω and $\mathbf{k}$ are the wave frequency and wave vector, respectively, and $\mathbf{v}$ is the particle velocity. Nonresonant waves $(\Omega,\mathbf{K})$ are waves for which neither linear nor nonlinear wave-particle resonance occurs. That is, $\Omega-\mathbf{K}\cdot\mathbf{v}\neq 0$ and $\Omega-\omega-(\mathbf{K}-\mathbf{k})\cdot\mathbf{v}\neq 0$. An example of processes in which both resonant and nonresonant waves participate is the interaction of plasma particles with the (resonant) low-frequency ion-acoustic waves and the (nonresonant) high-frequency Langmuir waves [8,10]. Such mixed processes are relevant in wave conversions, turbulent transport, and other processes involving large differences in wave frequency. They are especially important in many laboratory, space, and astrophysical plasmas which are open systems involving external energy sources and sinks [8,10–13].

II. FORMULATION

A qualitative picture for the nonlinear interaction between a particle and nonresonant and resonant waves may be given in a manner analogous to the relation between Fermi acceleration and the classical quasilinear process [10]. We assume that there exist randomly distributed localized regions (of average size l and average separation L) of electric fields with constant magnitude E_0 but randomly distributed signs. That is, the electric field experienced by a particle passing through each of the isolated field regions is constant and the average field of the entire system is zero. A particle moving through such a system will encounter the positive and negative field regions with the same probability. For definitive-

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ness we assume positive particle charge ($q>0$) and positive initial particle velocity [$v(0)>0$]. Accordingly, in a positive field region it gains on average an amount of energy qE_0l, and in a negative field region it loses the same amount of energy. A crucial point is that when the particle passes a positive field region, its velocity is increased by approximately qE_0l/mv, and it will need on average a shorter time (t_+) to reach the next region containing electric field. In the opposite case, its traveling time (t_-) will be correspondingly increased. The average increase of the particle energy $\mathcal{E}$ moving in such a system is then

$$\frac{d\langle\mathcal{E}\rangle}{dt}=\left\langle\frac{qE_0l}{t_+}-\frac{qE_0l}{t_-}\right\rangle$$
$$=\left\langle\frac{qE_0l}{L}\left(v+\frac{qE_0l}{mv}\right)-\frac{qE_0l}{L}\left(v-\frac{qE_0l}{mv}\right)\right\rangle$$
$$=\left\langle\frac{2q^2E_0^2l^2}{mvL}\right\rangle=\frac{2q^2L}{mv}\overline{E_0^2}, \quad (1)$$

where $\overline{E_0^2}=\langle E_0^2l^2/L^2\rangle$ is the mean-square electric field of the system. This energy increase is similar to that from quasilinear resonant wave-particle interaction [6,8].

To include the plasma-maser effect in the present model, it is necessary to simulate the interaction of the nonresonant particles with the resonant and nonresonant waves [8,9]. Accordingly, in addition to the resonant field regions we include in the system also a nonresonant (e.g., higher frequency) oscillating electric field. For simplicity we examine the case when there is an integer multiple of the wavelength of the nonresonant field, so that over the length l the particle will have an odd number of oscillations in the latter field. In the presence of such a field the particle velocity will increase and due to the Doppler effect the particle may not have an odd number of oscillations (over the length l). When the particle leaves the resonant field region its energy will be slightly less (or slightly more, depending on the sign of the resonant field) than qE_0l. In a sufficiently weak nonresonant wave field this difference is proportional to the field strength E of the nonresonant field. In other words, we have $\Delta\mathcal{E}_\pm=qE_0l(1\mp\alpha E)$, where α is a constant. Thus the average change of energy is

$$\frac{d\langle\mathcal{E}\rangle}{dt}=\left\langle\frac{\Delta\mathcal{E}_+}{L}\left(v+\frac{\Delta\mathcal{E}_+}{mv}\right)-\frac{\Delta\mathcal{E}_-}{L}\left(v-\frac{\Delta\mathcal{E}_-}{mv}\right)\right\rangle$$
$$=\frac{2q^2L}{mv}\overline{E_0^2}(1+\alpha^2\langle E^2\rangle), \quad (2)$$

so that there exists an additional contribution to the average change of the particle energy due to the presence of the nonresonant field. Note that this process is nonlinear (instead of quasilinear) in the sense that the effect is proportional to $\langle E_0^2\rangle\langle E^2\rangle$.

III. A FERMI-LIKE MODEL

In the corresponding mechanical model the particle receives kicks not from the distributed electric fields but from its bouncing (mirror reflecting) between two walls (1 and 2) oscillating with amplitudes $A_{1,2}$ and frequencies $\omega_{1,2}$ (Fig. 1).

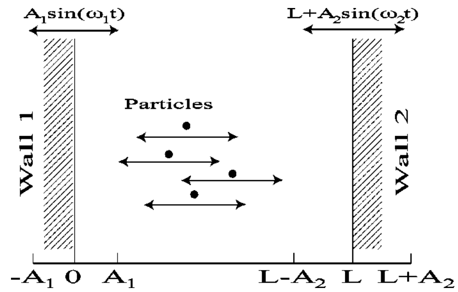

FIG. 1. The two-wall pinball model: one-dimensional simulation configuration.

In the classical Fermi-Ulam problem only one wall moves, and the setup has been used successfully to model the classical quasilinear process [3,8]. The mapping for the two-wall problem here is somewhat more complicated. A particle located at X_n at t_n and moving towards (the right) wall 2 will have its velocity mapped to

$$u_{n+1}=u_n-AF_2'(\Omega\Theta_n^{c2}), \quad (3)$$

$$\Theta_{n+1}=\Theta_n+\frac{2\pi M-x_n+AF_2(\Omega\Theta_n^{c2})}{2u_n}+\frac{2\pi M-x_n+AF_2(\Omega\Theta_n^{c2})}{2u_{n+1}}, \quad (4)$$

after completion of its collision with wall 2. Here $A=A_2/A_1$, $\Omega=\omega_2/\omega_1$, and the prime denotes derivative with respect to the argument. The other notations follow that commonly used in the Fermi-Ulam problem [3]: $\Theta_n=\omega_1t_n$, $u_n=v_n/2A_1\omega_1$, $M=L/2\pi A_1$, $x_n=X_n/A_1$, $F_2(\Theta)=\sin(\Theta)$, and $\Theta_n^{c2}=\Theta_n+[2\pi M-x_n+AF_2(\Omega\Theta_n^{c2})]/2u_n$ is the time when the particle collides with wall 2. Note that the normalizations are with respect to the parameters of wall 1.

Similarly, after the subsequent collision with (the left) wall 1, we may write

$$u_{n+2}=u_{n+1}-F_1'(\Theta_{n+1}^{c1}), \quad (5)$$

$$\Theta_{n+2}=\Theta_{n+1}+\frac{x_{n+1}+F_1(\Theta_{n+1}^{c1})}{2u_{n+1}}+\frac{x_{n+1}+F_1(\Theta_{n+1}^{c1})}{2u_{n+2}}, \quad (6)$$

where $\Theta_{n+1}^{c1}=\Theta_{n+1}+[x_{n+1}+F_1(\Theta_{n+1}^{c1})]/2u_{n+1}$ and $F_1(\Theta)=\sin(\Theta)$. The trajectory of the particle can thus be followed numerically by similar mappings. In the case of a fixed second wall ($A_2=0$), the standard mapping [3] for the classical Fermi problem can be recovered from Eqs. (5) and (6).

IV. NUMERICAL SIMULATION

In the numerical computation, we start with a given distribution of particles and velocities. That is, the model represents a statistical description of a generalized Fermi system. The equations of motion are solved using variable time steps to obtain the time evolution of the energy of each particle as well as the velocity distribution function. The transient and asymptotic states of the energy distribution of the particles are examined for different frequencies and amplitudes of the wall oscillations.

For the collision of particles with the walls we consider several cases, including the possibility of multiple collisions (e.g., when the particle velocity is small compared with the velocity of the wall, a similar problem was also discussed by Brahic [14]). The following parameters are used: the length of the system is $L=1$, the number of particles is $N=1000$–$10\,000$, and we take 50 as the number of output distributions. That is, if $t_{\rm tot}$ (typically $t_{\rm tot}=10^4$–10^5) is the total calculation time, the distributions are sampled in 50 time steps t_i ($i=1$–50) and averaged over $\Delta t=t_{\rm tot}/50$. Note that Δt is not the same as the time step for the integration of the equations of motion; it is the time period for averaging the particle distribution function.

A wide range of amplitudes and periods of wall oscillations and initial particle velocities is examined. The particles are assumed to be initially located at $X^j_{\rm init}=A_1+(L-A_1-A_2)(j-0.5)/N$, where j $(=1,\ldots,N)$ denotes the jth particle and $v^j_{\rm init}=(-1)^j v_{\rm init}(V_1+V_2)/2$ is its initial velocity, where $V_{1,2}=\omega_{1,2}A_{1,2}$ are the velocity amplitudes of the walls 1 and 2, and for convenience an ''initial velocity'' parameter $v_{\rm init}$ has been introduced. Typically, for a time span of $t_{\rm tot}=10\,000$, corresponding to the output averaging time $\Delta t=200$, a particle with an initial velocity of $v_{\rm init}=5$ will have bounced off the walls a few thousand times. In the following the subscript j shall be dropped when we discuss a typical particle.

First, we consider the case of very small oscillation amplitude of the second wall: $A_2=10^{-5}$ and $A_1=0.1$. The periods $T_{1,2}=2\pi/\omega_{1,2}$ of the wall oscillations are $T_1=0.05\pi$ and $T_2=7.14\times10^{-3}\pi$. The velocity amplitudes of the walls are then $V_1=2$ and $V_2=0.014$. The effect of the second wall is thus negligible, and we essentially obtain an evolution similar to that for the classical Fermi problem (Fig. 2). The energy of a particle oscillates with time but after an initial or transient quasilinear growth it becomes constant on the average. It is found that for relatively small initial particle velocities, a plateau eventually appears in the particle energy (momentum) distribution [Fig. 2(a)], similar to the well-known quasilinear energy redistribution in plasmas [4,6]. For larger initial velocities, a change of the character of the distribution function at high velocities occurs. This is associated with the formation of stochastic islands and a qualitative change of the particle orbits in the phase space [Fig. 2(b)]. The threshold initial velocity (obtained numerically) is $u_{\rm init}\approx 2.75$ and the velocity above which the distribution changes its character [note the sharp peak of the distribution function in Fig. 2(b)] is $u\approx 9$, agreeing with that from the standard Fermi mapping [3].

We note that even for smaller initial velocities long-time quasilinear evolution usually leads to the formation of stochastic islands and invariant curves [3]. However, in our model diffusion is very weak so that for smaller initial velocities the system does not have time to evolve into that regime. That is why in Fig. 2(a) we see only the quasilinear plateau, corresponding to purely stochastic particle motion without strong correlations. For larger initial velocities, the corresponding transient times are smaller and we obtain a qualitative change of the distribution function at higher velocities [Fig. 2(b)]. For still larger initial velocities, the particle motion rapidly goes into the strong correlation regime and no quasilinear-type diffusion is evident in the resulting distribution function [Fig. 2(c)].

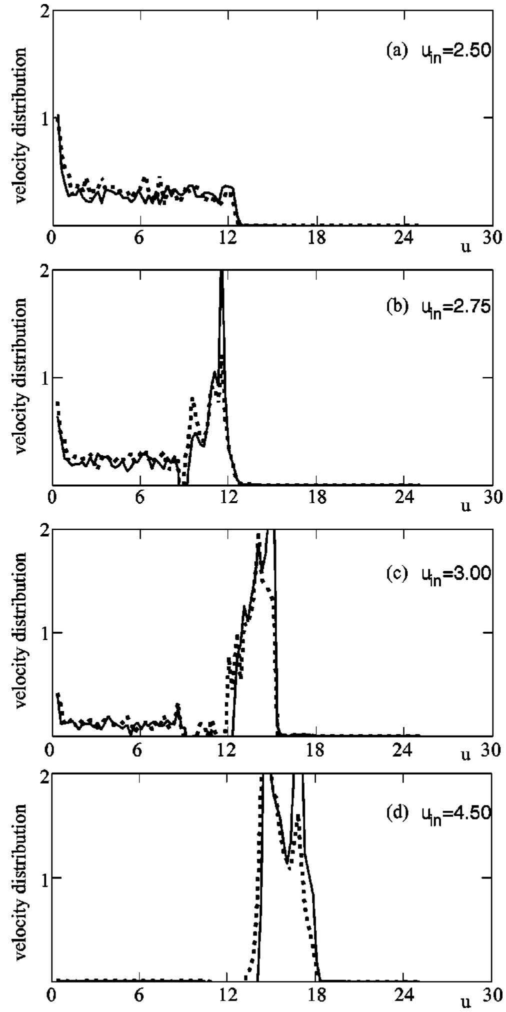

FIG. 2. The particle distribution function averaged at the 4th (dots) and 50th (solid lines) averaging intervals for different initial particle velocities and very small amplitudes of second-wall vibration: $A_1=0.1$ and $A_2=10^{-5}$.

When the second wall oscillates at higher amplitudes, the property of the resulting distribution function changes strongly. For $u_{\rm init}=3$ and $A_2=10^{-3}$–10^{-2} we could not find any end of the quasilinear evolution regime. Instead, particles are accelerated to much higher velocities without entering any strong-correlation regime in the phase space. In Figs. 3(b)–3(d), the appearance of high-energy particles in the absence of strong correlations is evident. There is also no stochastic island nor invariant curve in the phase space. We can therefore conclude that the presence of the second oscillating wall leads to a destruction of correlations for a much wider parameter range than in the case of a single oscillating

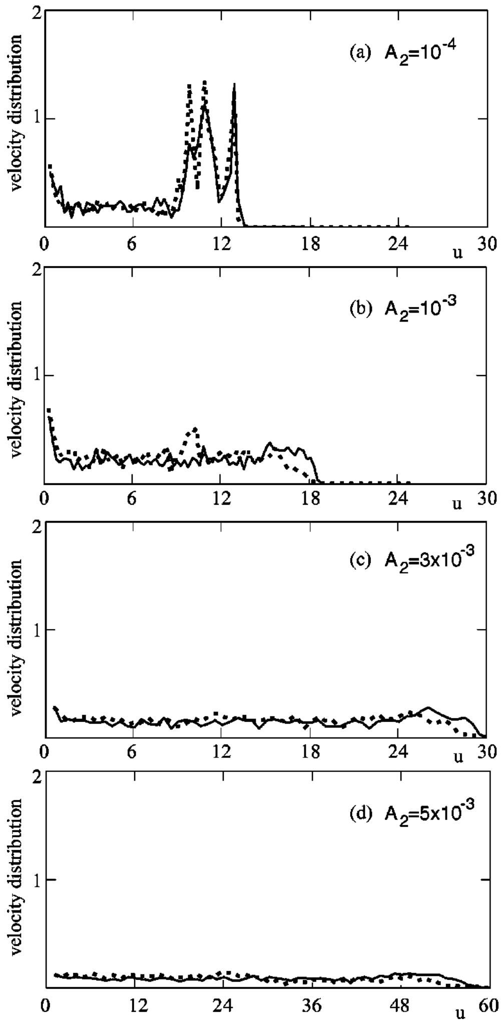

FIG. 3. The particle distribution function averaged at the 4th (dots) and 50th (solid lines) averaging intervals for higher amplitudes of second-wall vibration, with $A_1=0.1$ and $u_{\text{init}}=3$.

wall, as demonstrated by the lack of other qualitative changes beside the appearance of high-energy tails in the distribution function for the cases in Figs. 3(b)–3(d), as opposed to that of Fig. 3(a). In this case particles can be accelerated to very high energies.

V. DISCUSSION

Our results qualitatively agree with that of the analytical theory of the plasma-maser effect [10], although in the latter only a few terms were included in the nonlinear expansion, similar to the estimation given in Eq. (2). In particular, high-energy tails in the distribution function are formed because of a modification of quasilinear diffusion when the interaction of the nonresonant particles with the resonant and nonresonant waves (corresponding to the effect of the second oscillating wall in our model) is included. Moreover, the initial stage of the quasilinear evolution is also affected by the nonlinear modulation, as exhibited by the change in the length of the transient period of stochastic diffusion when the second oscillating wall is added. In fact, according to the theory [10,13], the nonlinear terms proportional to $\langle V_1^2\rangle\langle V_2^2\rangle$ are responsible for the change of character in the quasilinear evolution.

On the other hand, the result that the second oscillating wall can efficiently destroy correlations in the particle dynamics and thereby significantly change the character of the particle distribution at higher energies could not be predicted by any simple theory based on perturbation (in the amplitude of the waves) methods and the random-phase approximation [10,13].

It should be emphasized that since the motion of the walls is fixed in the present model, the self-consistent nature of the plasma waves and their evolution are in general not covered by the Fermi model. That is, the simulation here concentrates on the direct wave-particle interactions, important to both the quasilinear and plasma-maser effects. Furthermore, interaction among the particles, or generation and loss due to particle-particle collisions, are precluded. In dense low-temperature plasmas such collisions can dominate the purely dynamical phase randomization process studied here. However, inclusion of these processes would change the physical nature of the problem, in particular with respect to the analogy to the classical Fermi process, so that a separate consideration would be required.

ACKNOWLEDGMENTS

S.A.M. and S.V.V. were partly supported by the Australian Research Council and the Alexander von Humboldt Foundation. M.M.Y. was partly supported by the Sonderforschungsbereich 191.

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